Functional correctness specifications for concurrent data structures

Logical Atomicity in Iris

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Functional correctness of a queue

$\{ \quad \}$

$\texttt{enqueue}(q, v)$

$\{ \quad \}$

$\{ \quad \}$

$\texttt{dequeue}(q)$

$\{ \quad \}$

Functional correctness of a queue

$\text{Queue}(q, l)$:
q points to a queue storing list of values l

$$\{\text{Queue}(q, l)\}$$
$$\quad \texttt{enqueue}(q, v)$$
$$\{\text{Queue}(q, l +\!\!+ [v])\}$$

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$$\{\text{Queue}(q, l)\}\quad \texttt{enqueue}(q, v)\quad \{\text{Queue}(q, l + [v])\}$$

$$\{\text{Queue}(q, v :: l)\}\quad \texttt{dequeue}(q)\quad \{w.\, w = v * \text{Queue}(q, l)\}$$

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Binder for return value.

Specifying a sequential queue
is easy.

What about a concurrent queue?

Reuse the sequential specification?

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$$\{\mathrm{Queue}(q, v :: l)\}$$
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$$\{w.\, w = v * \mathrm{Queue}(q, l)\}$$

Reuse the sequential specification?

Precondition consumes queue ownership. Impossible to call `enqueue` or `dequeue` concurrently with other queue operations.

$$\{\mathrm{Queue}(q, l)\}\ \ \mathtt{enqueue}(q, v)\ \ \{\mathrm{Queue}(q, l +\!\!+ [v])\}$$

$$\{\mathrm{Queue}(q, v :: l)\}\ \ \mathtt{dequeue}(q)\ \ \{w.\ w = v * \mathrm{Queue}(q, l)\}$$

Reuse the sequential specification?

Precondition consumes queue ownership.

Even a non-thread-safe queue would satisfy this specification.

$\{\text{Queue}(q, l ++ [v])\}$ $\{w.\, w = v * \text{Queue}(q, l)\}$

Common solution:

- Use contextual refinement as spec
- Use linearizability to prove it

$$\frac{\frac{\texttt{enqueue} \precsim \texttt{seq\_enqueue\_locked}}{???}}{\{P\}\ \texttt{client}[\texttt{enqueue}]\ \{Q\}}$$

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What about heap accesses?

These look just as sequential:

$$\{\ell \mapsto v\}\ !\,\ell\ \{w.\ w = v * \ell \mapsto v\}$$

$$\{\ell \mapsto v\}\ \ell \leftarrow w\ \{\ell \mapsto w\}$$

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These look just as sequential:

$$\{\ell \mapsto v\} \, ! \, \ell \, \{w. \, w = v * \ell \mapsto v\}$$

$$\{\ell \mapsto v\} \, \ell \leftarrow w \, \{\ell \mapsto w\}$$

Concurrent use is possible thanks to the invariant rule!

What about heap accesses?

Invariant Rule

$$\frac{\{P * I\}\ \ell \leftarrow w\ \{Q * I\}}{\boxed{I} \vdash \{P\}\ \ell \leftarrow w\ \{Q\}}$$

(consider $I \triangleq \exists v.\ \ell \mapsto v$)

Invariant rule!

What about heap accesses?

Invariant Rule

$$\frac{\{P * I\}\; e\; \{Q * I\} \qquad \text{phy\_atomic(e)}}{\boxed{I} \vdash \{P\}\; e\; \{Q\}}$$

Invariant rule!

An operation is atomic if we can open invariants around it.

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Logical atomicity

lets us open invariants around non-physically-atomic operations.

Outline

1. How to specify and use basic logically atomic operations in Iris
2. Advanced logically atomic patterns: aborting, helping
3. Summary and case studies

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Logically Atomic Queue

Logically Atomic Hoare triples inspired by TaDA

(Pinto, Dinsdale-Young, Gardner; 2014)

$$\langle l.\, \mathrm{Queue}(q, l) \rangle$$
$$\quad \mathtt{enqueue}(q, v)$$
$$\langle \mathrm{Queue}(q, l \mathbin{+\!\!+} [v]) \rangle$$

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$$\langle l.\ \mathsf{Queue}(q, l)\rangle$$
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$$\langle v.\ v = \mathsf{head}(l) * \mathsf{Queue}(q, \mathsf{tail}(l))\rangle$$

Logically Atomic Queue

Logically Atomic Hoare triples inspired by TaDA
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l is picked at the linearization point

$$\langle l.\ \mathsf{Queue}(q, l)\rangle$$
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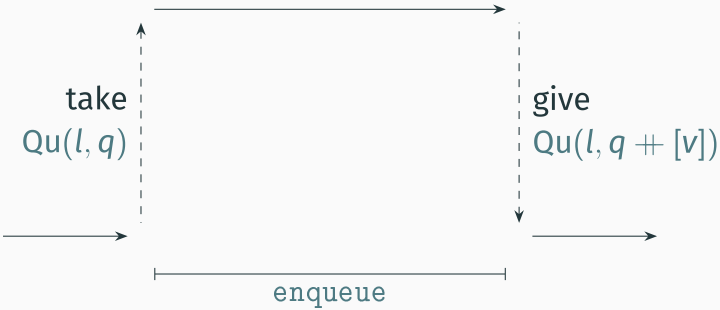
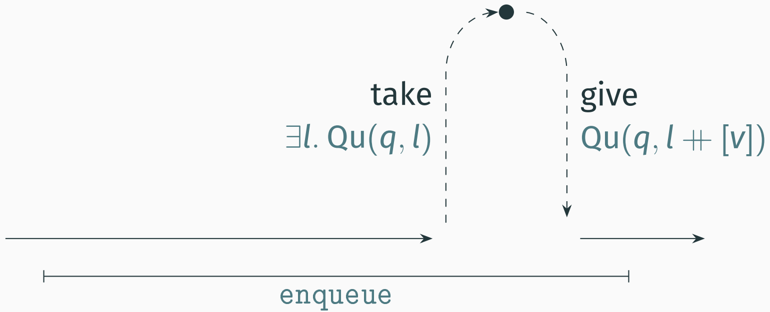
$$\forall l.\ \{\mathrm{Qu}(q, l)\}\ \texttt{enqueue}(q, v)\ \{\mathrm{Qu}(q, l +\!\!+ [v])\}$$



$\mathsf{IsQu}(q) \vdash \langle l.\ \mathsf{Qu}(q, l) \rangle\ \texttt{enqueue}(q, v)\ \langle \mathsf{Qu}(q, l \mathbin{+\!\!+} [v]) \rangle$



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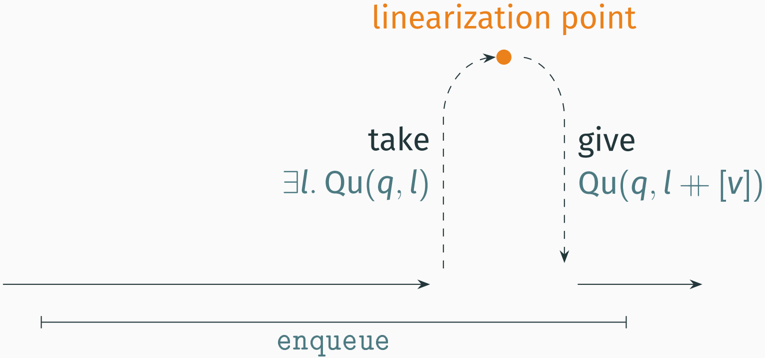

$$\mathsf{IsQu}(q) \vdash \langle l.\, \mathsf{Qu}(q, l)\rangle\ \texttt{enqueue}(q, v)\ \langle \mathsf{Qu}(q, l \mathbin{+\!\!+} [v])\rangle \triangleq$$

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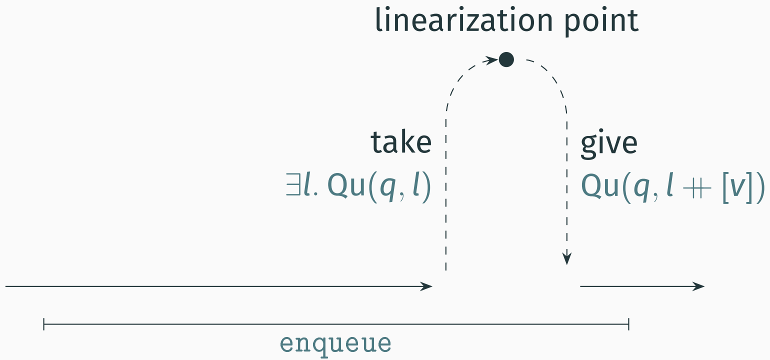

Interlude: Masks and view shifts in Iris

$$\frac{\{P * I\}\; e\; \{Q * I\} \qquad \text{phy\_atomic(e)}}{\boxed{I} \;\vdash \{P\}\; e\; \{Q\}}$$

Interlude: Masks and view shifts in Iris

$$\frac{\{P * I\}\; e\; \{Q * I\}_{\mathcal{E}\setminus\mathcal{N}} \qquad \mathcal{N} \subseteq \mathcal{E} \qquad \text{phy\_atomic(e)}}{\boxed{I}^{\mathcal{N}} \vdash \{P\}\; e\; \{Q\}_{\mathcal{E}}}$$

Namespace of the invariant

Mask of the Hoare triple

Interlude: Masks and view shifts in Iris

View shift: "linear ghost step" (without code)

$$\frac{P * I \equiv\!\!\ast_{\mathcal{E}\setminus\mathcal{N}} Q * I \qquad \mathcal{N} \subseteq \mathcal{E}}{\boxed{I}^{\mathcal{N}} \vdash P \equiv\!\!\ast_{\mathcal{E}} Q}$$

Namespace of the invariant

Mask of the view shift

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$$\frac{P * I \Rrightarrow_{\mathcal{E} \setminus \mathcal{N}} Q * I \qquad \mathcal{N} \subseteq \mathcal{E}}{\boxed{I}^{\mathcal{N}} \vdash P \Rrightarrow_{\mathcal{E}} Q}$$

$P_1 \Rrightarrow_{\mathcal{E}} P_2$: view shift from P_1 to P_2
using only invariants $\mathcal{N} \subseteq \mathcal{E}$

Interlude: Masks and view shifts in Iris

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$P_1 \;{}^{\mathcal{E}_1}{\Rrightarrow}^{\mathcal{E}_2}\; P_2$: mask-changing view shift from P_1 to P_2

Interlude: Mask-changing view shifts

$$\frac{\mathcal{E} \subseteq \mathcal{N}}{\boxed{I}^{\mathcal{N}} \vdash P \; {}^{\mathcal{E}}{\equiv\!\!\!\ast}^{\mathcal{E}\setminus\mathcal{N}} \; P * I}$$

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Interlude: Mask-changing view shifts

$$\frac{\mathcal{E} \subseteq \mathcal{N}}{\boxed{I}^{\mathcal{N}} \vdash P \;{}^{\mathcal{E}}{\Rrightarrow}\!\!\ast^{\mathcal{E}\setminus\mathcal{N}}\; P * I} \qquad \frac{\mathcal{E} \subseteq \mathcal{N}}{\boxed{I}^{\mathcal{N}} \vdash Q * I \;{}^{\mathcal{E}\setminus\mathcal{N}}{\Rrightarrow}\!\!\ast^{\mathcal{E}}\; Q}$$

$$\frac{\begin{array}{c}\text{phy\_atomic(e)} \\ P \;{}^{\mathcal{E}}{\Rrightarrow}\!\!\ast^{\mathcal{E}'}\; P' \qquad \{P'\}\; e \;\{Q'\}_{\mathcal{E}'} \qquad Q' \;{}^{\mathcal{E}'}{\Rrightarrow}\!\!\ast^{\mathcal{E}}\; Q\end{array}}{\{P\}\; e \;\{Q\}_{\mathcal{E}}}$$

Interlude: Mask-changing view shifts

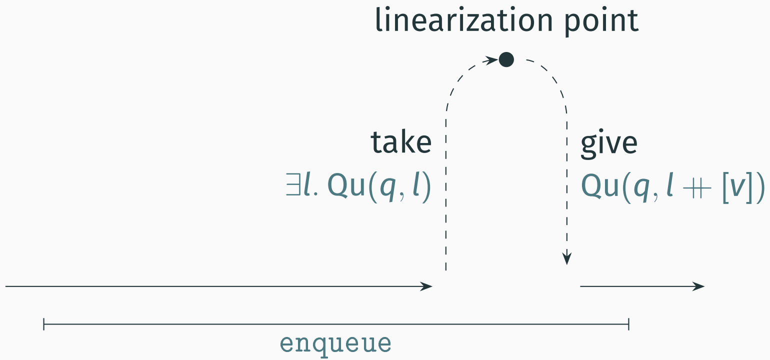
$$\frac{\mathcal{E} \subseteq \mathcal{N}}{\boxed{I}^{\mathcal{N}} \vdash P \mathrel{{}^{\mathcal{E}}\!\!\Rrightarrow\!\!\ast^{\mathcal{E}\setminus\mathcal{N}}} P * I} \qquad \frac{\mathcal{E} \subseteq \mathcal{N}}{\boxed{I}^{\mathcal{N}} \vdash Q * I \mathrel{{}^{\mathcal{E}\setminus\mathcal{N}}\!\!\Rrightarrow\!\!\ast^{\mathcal{E}}} Q}$$

$$\frac{P \mathrel{{}^{\mathcal{E}}\!\!\Rrightarrow\!\!\ast^{\mathcal{E}'}} P' \qquad \begin{array}{c}\text{phy\_atomic(e)}\\ \{P'\}\, e\, \{Q'\}_{\mathcal{E}'}\end{array} \qquad Q' \mathrel{{}^{\mathcal{E}'}\!\!\Rrightarrow\!\!\ast^{\mathcal{E}}} Q}{\{P\}\, e\, \{Q\}_{\mathcal{E}}}$$

Together, these three rules imply the invariant open rule!

$$\mathsf{IsQu}(q) \vdash \langle l.\ \mathsf{Qu}(q, l)\rangle\ \texttt{enqueue}(q, v)\ \langle \mathsf{Qu}(q, l \mathbin{+\!\!+} [v])\rangle \triangleq$$

$$\forall R.\ \{\mathsf{AU}_R\}\ \texttt{enqueue}(q, v)\ \{R\}$$



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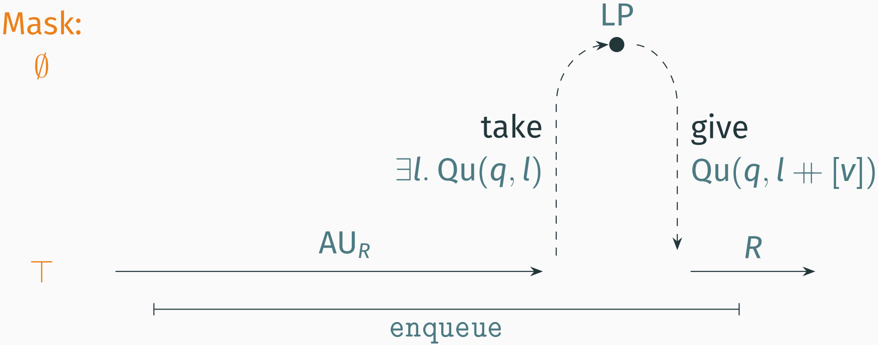

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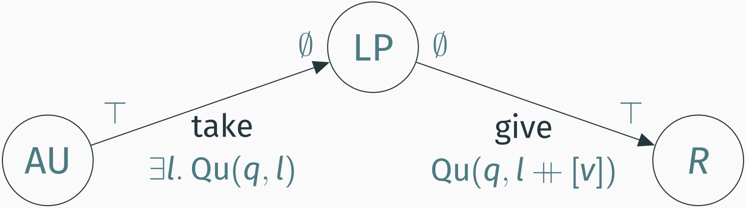
Meaning of atomic update AU_R:



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$\mathsf{AU}_R \triangleq$

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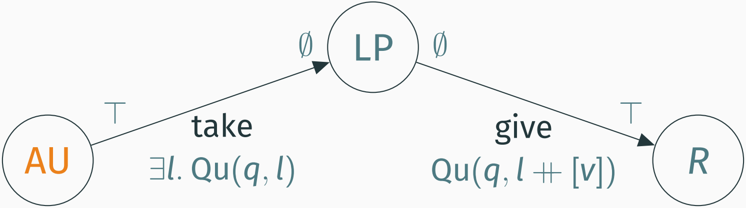

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$$\mathsf{AU}_R \triangleq \mathsf{True}\ {}^{\top}\!\!\Rrightarrow\!\!\ast^{\emptyset}$$

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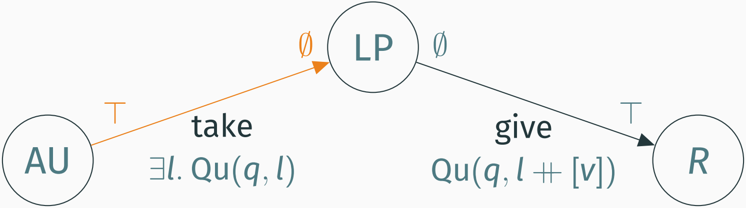

$$\forall R.\ \{\mathsf{AU}_R\}\ \texttt{enqueue}(q, v)\ \{R\}$$

$$\mathsf{AU}_R \triangleq \mathsf{True}\ {}^{\top}\!\!\Rrightarrow\!\!\ast^{\emptyset}\ \exists l.\ \mathsf{Qu}(q, l)$$

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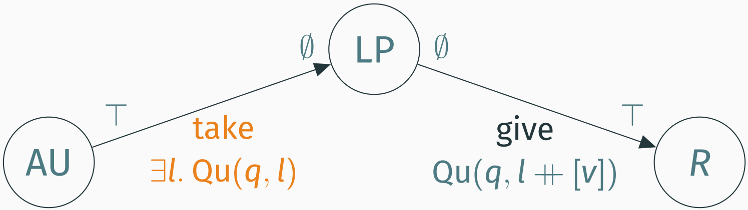

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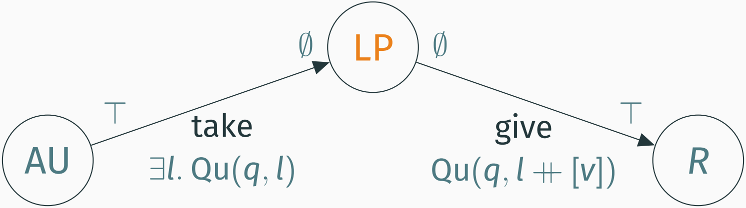

$$\forall R.\ \{\mathsf{AU}_R\}\ \mathtt{enqueue}(q, v)\ \{R\}$$

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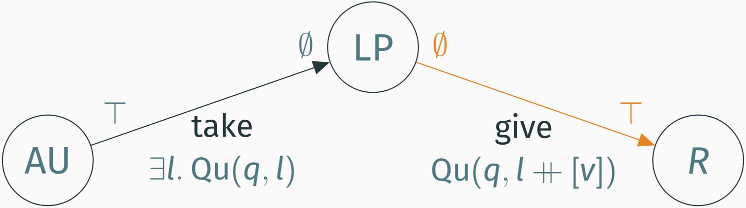
$$\mathsf{LP} \triangleq \qquad\qquad\qquad {}^{\emptyset}\!\Rrightarrow\!\!*^{\top}$$

Meaning of atomic update AU_R:



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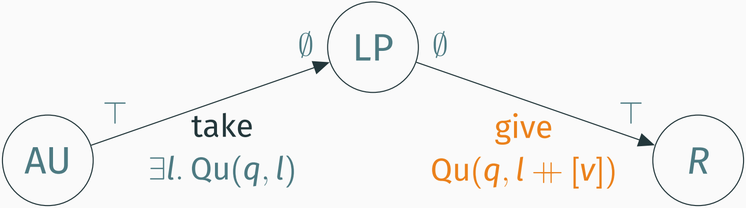

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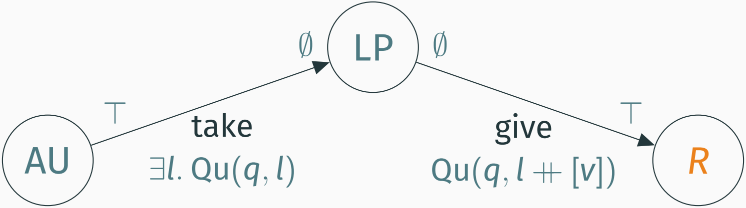

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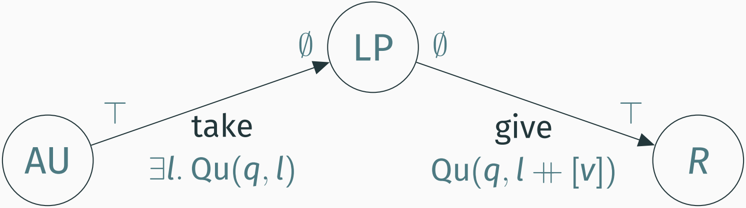

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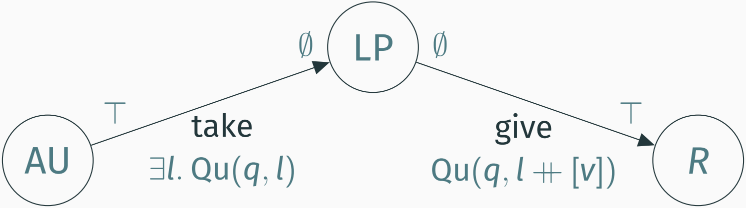

$$\forall R.\ \{\mathsf{AU}_R\}\ \texttt{enqueue}(q, v)\ \{R\}$$

Specification of logically atomic `enqueue`:

$$\mathsf{IsQu}(l) \vdash \langle l.\ \mathsf{Qu}(q, l)\rangle\ \texttt{enqueue}(q, v)\ \langle \mathsf{Qu}(q, l +\!\!+ [v])\rangle$$

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Specification of logically atomic `enqueue`:

$$\mathsf{IsQu}(l) \vdash \langle l.\ \mathsf{Qu}(q, l)\rangle\ \mathtt{enqueue}(q, v)\ \langle \mathsf{Qu}(q, l +\!\!+ [v])\rangle$$

which expands to:

$$\mathsf{IsQu}(l) \vdash \forall R.\ \{\mathsf{AU}_R\}\ \mathtt{enqueue}(q, v)\ \{R\}$$

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How do we use a logically atomic triple?

Let's say we have a shared queue that contains only even numbers:

$$\exists l.\, \mathsf{Qu}(q, l) * \forall n \in l.\, \mathsf{even}(n)$$

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How can we `enqueue` and `dequeue` on it?

`enqueue` on a shared queue

Assume: $\mathsf{IsQu}(q)$, $\boxed{\exists l.\, \mathsf{Qu}(q, l) * \forall n \in l.\, \mathsf{even}(n)}^{\mathcal{N}}$

Goal: $\{\mathsf{True}\}\ \texttt{enqueue}(q, 2)\ \{\mathsf{True}\}$

`enqueue` on a shared queue

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Remember that we have:

$$\mathsf{IsQu}(l) \vdash \forall R.\, \{\mathsf{AU}_R\}\ \mathtt{enqueue}(q, 2)\ \{R\}$$

So it suffices to show:

$$\boxed{\exists l.\, \mathsf{Qu}(q, l) * \forall n \in l.\, \mathsf{even}(n)}^{\mathcal{N}} \vdash \mathsf{AU}_{\mathsf{True}}$$

`enqueue` on a shared queue

Assume: $\boxed{\exists l.\, \mathrm{Qu}(q, l) * \forall n \in l.\, \mathrm{even}(n)}^{\mathcal{N}}$

Goal:

$\mathrm{AU}_{\mathrm{True}}$

`enqueue` on a shared queue

Assume: $\boxed{\exists l.\, \mathrm{Qu}(q, l) * \forall n \in l.\, \mathrm{even}(n)}^{\mathcal{N}}$

Goal:

$$\mathrm{True} \;{}^{\top}{\Rrightarrow\!\!*}^{\emptyset}\; \exists l.\, \mathrm{Qu}(q, l) * \left(\mathrm{Qu}(q, l +\!\!+ [2]) \;{}^{\emptyset}{\Rrightarrow\!\!*}^{\top}\; \mathrm{True}\right)$$

`enqueue` on a shared queue

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Goal:

$$\mathrm{True}\ {}^{\top}\!\Rrightarrow^{\emptyset} \exists l.\, \mathrm{Qu}(q, l) * \left(\mathrm{Qu}(q, l \mathbin{+\!\!+} [2])\ {}^{\emptyset}\!\Rrightarrow^{\top} \mathrm{True}\right)$$

The invariant rules give us:

$$\boxed{I}^{\mathcal{N}} \vdash \mathrm{True}\ {}^{\top}\!\Rrightarrow^{\emptyset} I * \left(I\ {}^{\emptyset}\!\Rrightarrow^{\top} \mathrm{True}\right)$$

for our $I \triangleq \exists l.\, \mathrm{Qu}(q, l) * \forall n \in l.\, \mathrm{even}(n)$

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Goal:

$\mathrm{True} \;{}^{\top}{\Rrightarrow}^{\emptyset}\; \exists l.\, \mathrm{Qu}(q, l) * \left(\mathrm{Qu}(q, l +\!\!+ [2]) \;{}^{\emptyset}{\Rrightarrow}^{\top}\; \mathrm{True}\right)$

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for our $I \triangleq \exists l.\, \mathrm{Qu}(q, l) * \forall n \in l.\, \mathrm{even}(n)$

Now all we need is

$I -\!\!* \; \exists l.\, \mathrm{Qu}(q, l) * \left(\mathrm{Qu}(q, l +\!\!+ [2]) -\!\!* \; I\right)$

which is trivial.

We have seen logical atomicity for `enqueue`.

This can be generalized!

$$\langle x.\, P(x)\rangle\; e\; \langle Q(x)\rangle_{\mathcal{E}} \triangleq \forall R.\, \{\mathsf{AU}_R\}\; e\; \{R\}$$

$$\mathsf{AU}_R \triangleq \mathsf{True}\ {}^{\top\setminus\mathcal{E}}{\Rrightarrow\!\!\ast}^{\emptyset}\ \exists x.\, P(x) * \left(Q(x)\ {}^{\emptyset}{\Rrightarrow\!\!\ast}^{\top\setminus\mathcal{E}}\ R\right)$$

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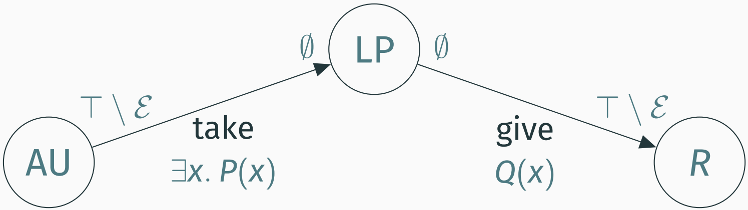

$$\langle x.\ P(x)\rangle\ e\ \langle Q(x)\rangle_{\mathcal{E}} \triangleq \forall R.\ \{\mathsf{AU}_R\}\ e\ \{R\}$$

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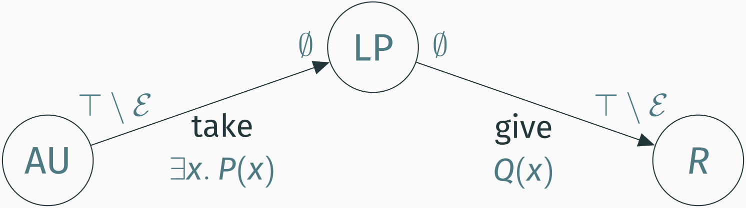
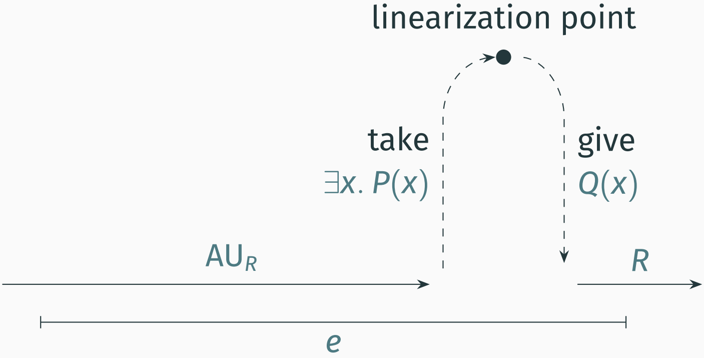
Meaning of atomic update AU_R:



$$\langle x.\, P(x)\rangle\ e\ \langle Q(x)\rangle_{\mathcal{E}} \triangleq \forall R.\ \{\mathrm{AU}_R\}\ e\ \{R\}$$

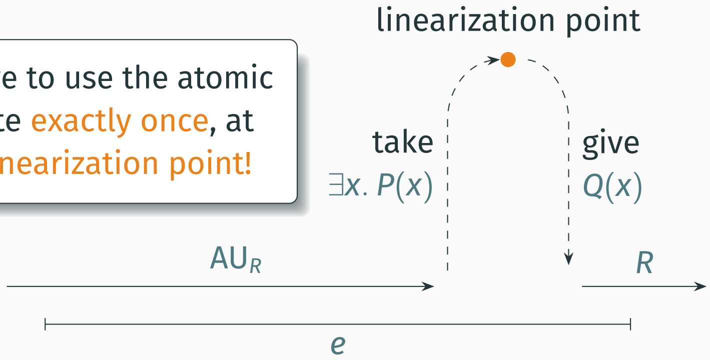
$$\mathrm{AU}_R \triangleq \mathrm{True}\ {}^{\top\setminus\mathcal{E}}\!\!\Rrightarrow\!\!\ast^{\emptyset}\ \exists x.\, P(x) \ast \left(Q(x)\ {}^{\emptyset}\!\!\Rrightarrow\!\!\ast^{\top\setminus\mathcal{E}}\ R\right)$$



$$\langle x.\, P(x)\rangle\; e\; \langle Q(x)\rangle_{\mathcal{E}} \triangleq \forall R.\, \{\mathrm{AU}_R\}\; e\; \{R\}$$

$$\mathrm{AU}_R \triangleq \mathrm{True}\; {}^{\top\setminus\mathcal{E}}{\Rrightarrow\!\!\ast}^{\emptyset}\; \exists x.\, P(x) * \left(Q(x)\; {}^{\emptyset}{\Rrightarrow\!\!\ast}^{\top\setminus\mathcal{E}}\; R\right)$$

We have to use the atomic update exactly once, at the linearization point!



Logically atomic triples enjoy the Invariant Rule:

$$\frac{\langle x.\, P * I \rangle\ e\ \langle Q * I \rangle_{\mathcal{E} \setminus \mathcal{N}} \qquad \mathcal{N} \subseteq \mathcal{E}}{\boxed{I}^{\mathcal{N}} \vdash \langle x.\, P \rangle\ e\ \langle Q \rangle_{\mathcal{E}}}$$

"An operation is atomic
if we can open invariants around it."

Outline

1. How to specify and use basic logically atomic operations in Iris
2. Advanced logically atomic patterns: aborting, helping
3. Summary and case studies

Can we specify and prove a blocking `dequeue`?

Implementation:

```
blocking_dequeue(q) ≜
    match dequeue(q) with
     Some(x) ⇒ x
    | None    ⇒ blocking_dequeue(q)
    end
```

Implementation:

$$\texttt{blocking\_dequeue}(q) \triangleq$$

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Specification:

$$\langle l.\ \mathsf{Queue}(q, l)\rangle$$
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expands to

$\forall R.\ \{\mathsf{AU}_R\}\ \texttt{blocking\_dequeue}(q)\ \{v.\ R(v)\}$ where

$$\mathsf{AU}_R \triangleq \mathsf{True}\ {}^{\top}{\Rrightarrow}\!\!\ast^{\emptyset}\ \exists l.\ \mathsf{Queue}(q, l) * \left(\forall v.\ (\exists l'.\ l = v :: l' * \mathsf{Queue}(q, l'))\ {}^{\emptyset}{\Rrightarrow}\!\!\ast^{\top}\ R(v)\right)$$

$$\langle x.\, P(x)\rangle\; e\; \langle Q(x)\rangle_{\mathcal{E}} \triangleq \forall R.\, \{\mathrm{AU}_R\}\; e\; \{R\}$$

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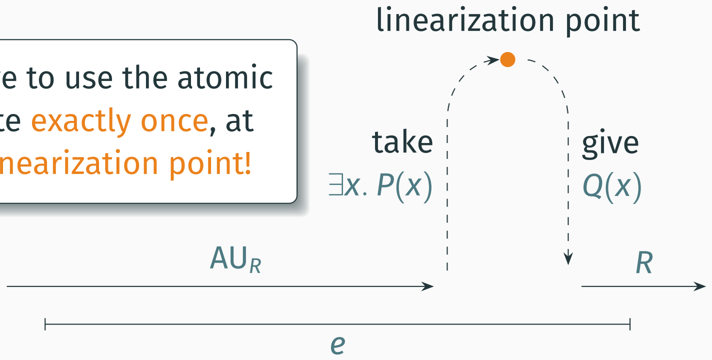

Implementation:

```
blocking_dequeue(q) ≜
    match dequeue(q) with
     Some(x) ⇒ x
    | None   ⇒ blocking_dequeue(q)
    end
```

The first call to `dequeue` will consume AU!

Implementation:

$\texttt{blocking\_dequeue}(q) \triangleq$

$\texttt{match dequeue}(q)\ \texttt{with}$

To be able to derive `blocking_dequeue` (without breaking the abstraction), we have to adjust our definition of logical atomicity.

consume AU!

Implementation:

```
blocking_dequeue(q) ≜
    match dequeue(q) with
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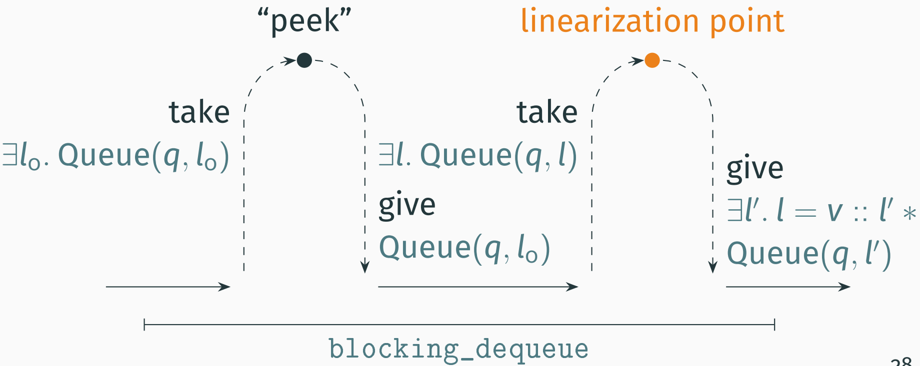

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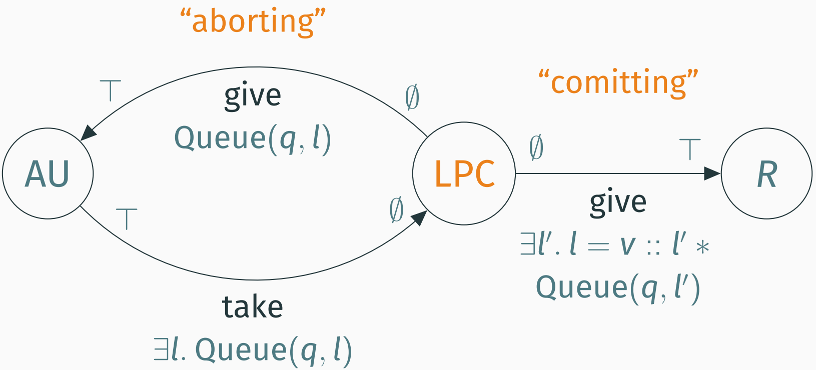
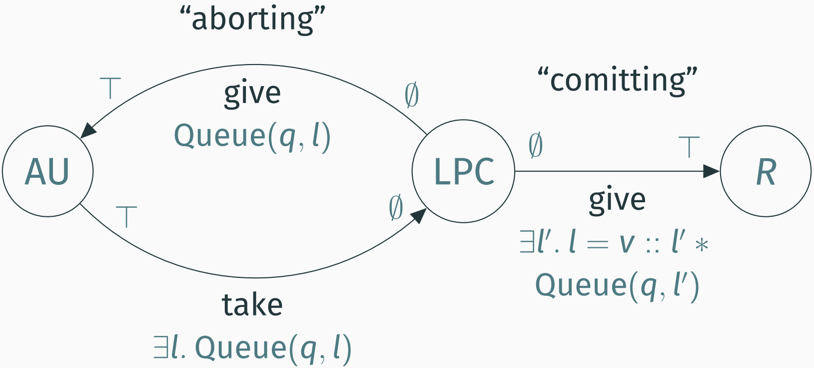


$$\mathsf{AU}_R \triangleq \mathsf{True} \;{}^{\top}\!\!\Rrightarrow\!\!\!\ast^{\emptyset}\; \exists l.\, \mathsf{Queue}(q, l) \ast \mathsf{LPC}_{R,l}$$

$$\mathsf{LPC}_{R,l} \triangleq$$

$$\left(\forall v.\, (\exists l'.\, l = v :: l' \ast \mathsf{Queue}(q, l')) \;{}^{\emptyset}\!\!\Rrightarrow\!\!\!\ast^{\top}\; R(v)\right)$$



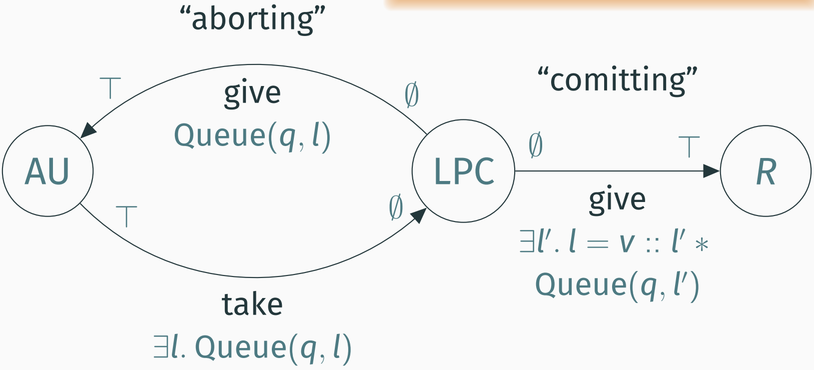
$$\mathrm{AU}_R \triangleq \mathrm{True} \;{}^{\top}\!\!\Rrightarrow\!\!\ast^{\emptyset}\; \exists l.\, \mathrm{Queue}(q, l) \ast \mathrm{LPC}_{R,l}$$

$$\mathrm{LPC}_{R,l} \triangleq$$

$$\wedge$$

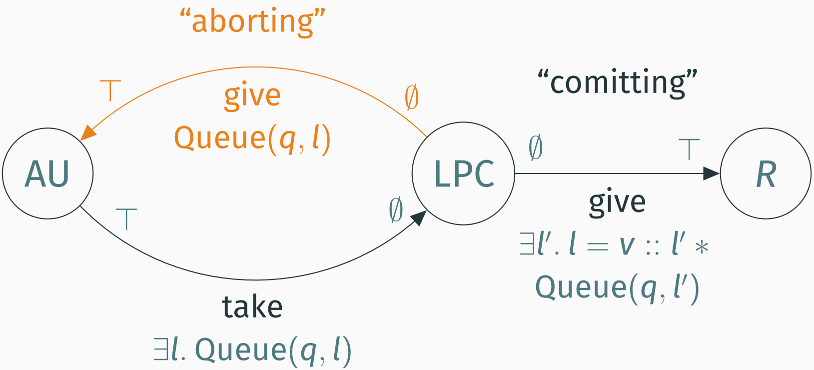
$$\Big(\forall v.\, \big(\exists l'.\, l = v :: l' \ast \mathrm{Queue}(q, l')\big) \;{}^{\emptyset}\!\!\Rrightarrow\!\!\ast^{\top}\; R(v)\Big)$$

Conjunction ≅ “choice”



$$\mathsf{AU}_R \triangleq \mathsf{True} \;{}^{\top}\!\!\Rrightarrow\!\!\ast^{\emptyset}\; \exists l.\, \mathsf{Queue}(q, l) * \mathsf{LPC}_{R,l}$$

$$\mathsf{LPC}_{R,l} \triangleq \left(\mathsf{Queue}(q, l) \;{}^{\emptyset}\!\!\Rrightarrow\!\!\ast^{\top}\; \mathsf{AU}_R\right) \wedge \left(\forall v.\, \left(\exists l'.\, l = v :: l' * \mathsf{Queue}(q, l')\right) \;{}^{\emptyset}\!\!\Rrightarrow\!\!\ast^{\top}\; R(v)\right)$$



$$\mathsf{AU}_R \triangleq \mathsf{True}\ {}^{\top}\!\!\Rrightarrow\!\!\ast^{\emptyset}\ \exists l.\ \mathsf{Queue}(q, l) * \mathsf{LPC}_{R,l}$$

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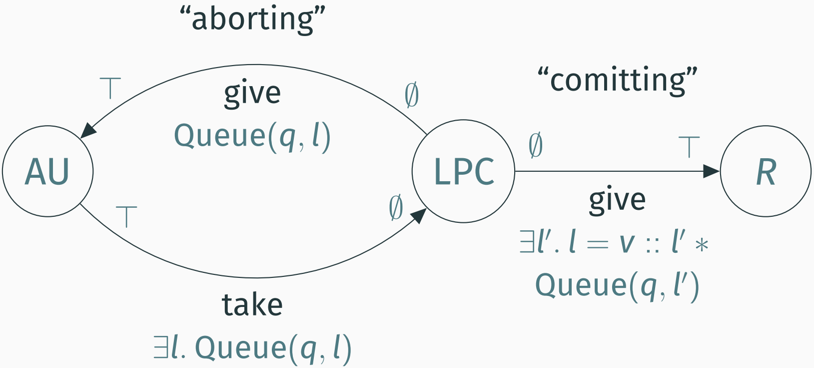


$$\mathsf{AU}_R \triangleq \mathsf{True} \;{}^{\top}\!\!\Rrightarrow\!\!\ast^{\emptyset}\; \exists l.\, \mathsf{Queue}(q, l) \ast \mathsf{LPC}_{R,l}$$

$$\begin{aligned}\mathsf{LPC}_{R,l} \triangleq\; & \left(\mathsf{Queue}(q, l) \;{}^{\emptyset}\!\!\Rrightarrow\!\!\ast^{\top}\; \mathsf{AU}_R\right) \wedge \\ & \left(\forall v.\, \left(\exists l'.\, l = v :: l' \ast \mathsf{Queue}(q, l')\right) \;{}^{\emptyset}\!\!\Rrightarrow\!\!\ast^{\top}\; R(v)\right)\end{aligned}$$

We can tie the recursive knot using a (greatest) fixed point.

By “aborting” when `dequeue` fails, we can prove the desired specification for `blocking_dequeue`.

$$\langle l.\ \mathrm{Queue}(q, l) \rangle$$
$$\quad \texttt{blocking\_dequeue}(q)$$
$$\langle v.\ \exists l'.\ l = v :: l' * \mathrm{Queue}(q, l') \rangle$$

Helping

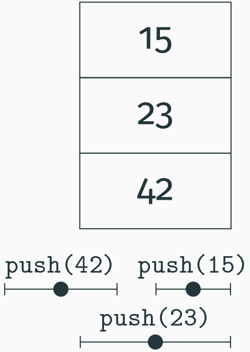
One thread can complete the action of another.

Helping

One thread can complete the action of another.

For example:

Stack content:



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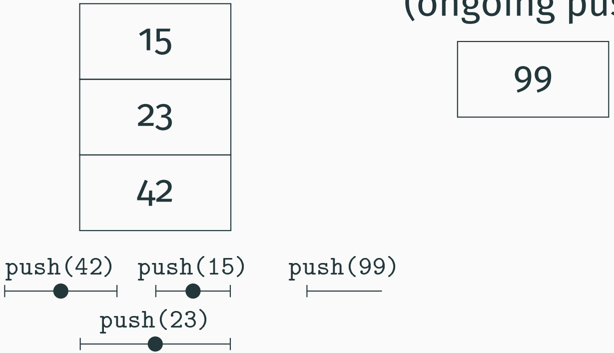
For example:

Stack content:

"Bypass offer":
(ongoing push)



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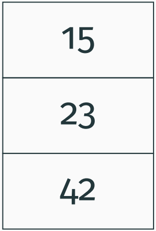

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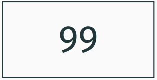

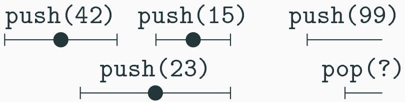

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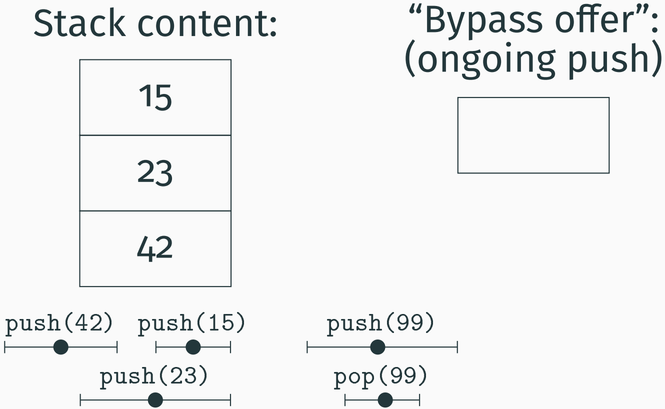

Helping

One thread can complete the action of another

AU_R is just a (separation logic) resource!

We can send it from one thread to another.

1. Thread A puts their AU_R into invariant
2. Thread B receives AU_R
3. Thread B completes both runs AU_R and its own $AU_{R'}$
4. Thread B puts results R back into invariant
5. Thread A obtains result R and completes

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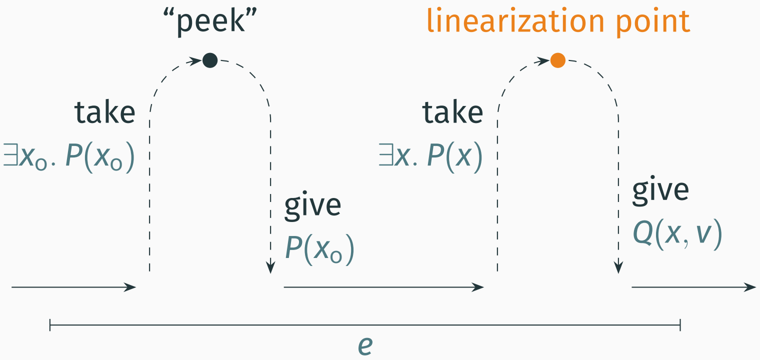

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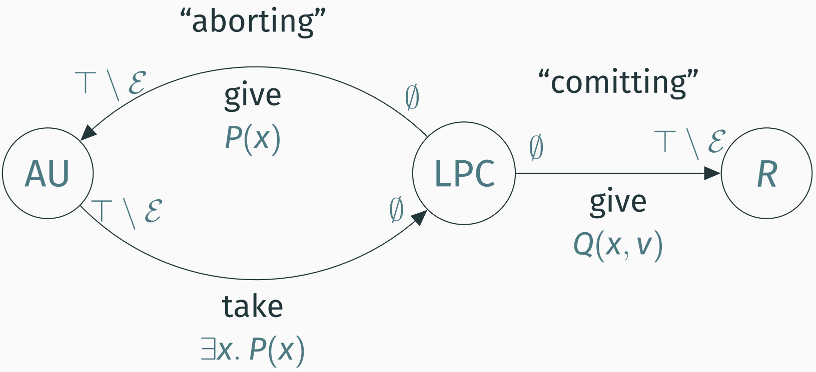

Logically Atomic Case Studies

- Elimination Stack on abstract heap

https://iris-project.org

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- Elimination Stack on abstract heap
- Flat Combiner (by Zhen)

Iris

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Iris

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- Herlihy-Wing-Queue
  (by Rodolphe, Derek, Gaurav)

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Logically Atomic Case Studies

- Elimination Stack on abstract heap

Logical atomicity **implies linearizability:**

"Theorems for Free from Separation Logic Specifications"

Birkedal, Dinsdale-Young, Guéneau, Jaber, Svendsen, Tzevelekos; ICFP 2021

https://iris-project.org

Logical Atomicity lets us give

- concise and powerful
- Hoare-style specifications
- to concurrent data structures
- that make use of helping.

https://iris-project.org